General Certificate of Education
June 2006
Advanced Level Examination

MATHEMATICS MM2B
Unit Mechanics 2B

Tuesday 6 June 2006   1.30 pm to 3.00 pm

For this paper you must have:
 • an 8-page answer book
 • the **blue** AQA booklet of formulae and statistical tables
You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
• Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
• Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MM2B.
• Answer **all** questions.
• Show all necessary working; otherwise marks for method may be lost.
• The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
• Take $g = 9.8\,\text{m s}^{-2}$, unless stated otherwise.

Information
• The maximum mark for this paper is 75.
• The marks for questions are shown in brackets.
• Unit Mechanics 2B has a **written paper only**.

Advice
• Unless stated otherwise, you may quote formulae, without proof, from the booklet.

MM2B

Answer **all** questions.

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1 A particle moves in a horizontal plane, in which the unit vectors **i** and **j** are directed east and north respectively. At time t seconds, its position vector, **r** metres, is given by

$$\mathbf{r} = (2t^3 - t^2 + 6)\mathbf{i} + (8 - 4t^3 + t)\mathbf{j}$$

   (a) Find an expression for the velocity of the particle at time t. *(3 marks)*

   (b)   (i) Find the velocity of the particle when $t = \frac{1}{3}$. *(2 marks)*

       (ii) State the direction in which the particle is travelling at this time. *(1 mark)*

   (c) Find the acceleration of the particle when $t = 4$. *(3 marks)*

   (d) The mass of the particle is 6 kg. Find the magnitude of the resultant force on the particle when $t = 4$. *(3 marks)*

2 A ball of mass 0.6 kg is thrown vertically upwards from ground level with an initial speed of $14 \, \mathrm{m \, s^{-1}}$.

   (a) Calculate the initial kinetic energy of the ball. *(2 marks)*

   (b) Assuming that no resistance forces act on the ball, use an energy method to find the maximum height reached by the ball. *(3 marks)*

   (c) An experiment is conducted to confirm the maximum height for the ball calculated in part (b). In this experiment the ball rises to a height of only 8 metres.

      (i) Find the work done against the air resistance force that acts on the ball as it moves. *(3 marks)*

      (ii) Assuming that the air resistance force is constant, find its magnitude. *(2 marks)*

   (d) Explain why it is **not** realistic to model the air resistance as a constant force. *(1 mark)*

3 The diagram shows a uniform rod, *AB*, of mass 10 kg and length 5 metres. The rod is held in equilibrium in a horizontal position, by a support at *C* and a light vertical rope attached to *A*, where *AC* is 2 metres.
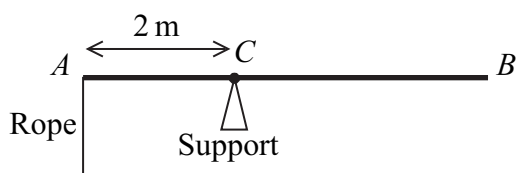

(a) Draw and label a diagram to show the forces acting on the rod. *(1 mark)*

(b) Show that the tension in the rope is 24.5 N. *(3 marks)*

(c) A package of mass *m* kg is suspended from *B*. The tension in the rope has to be doubled to maintain equilibrium.

   (i) Find *m*. *(4 marks)*

   (ii) Find the magnitude of the force exerted on the rod by the support. *(3 marks)*

(d) Explain how you have used the fact that the rod is uniform in your solution. *(1 mark)*

4 A particle of mass *m* is suspended from a fixed point *O* by a light inextensible string of length *l*. The particle hangs in equilibrium at the point *P* vertically below *O*. The particle is then set into motion with a horizontal velocity *U* so that it moves in a complete vertical circle with centre *O*. The point *Q* on the circle is such that $\angle POQ = 60°$, as shown in the diagram.
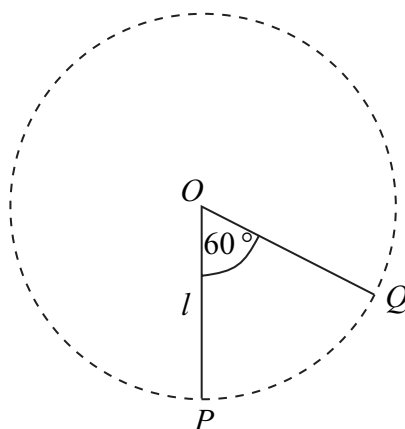
(a) Find, in terms of *g*, *l* and *U*, the speed of the particle at *Q*. *(4 marks)*

(b) Find, in terms of *g*, *l*, *m* and *U*, the tension in the string when the particle is at *Q*. *(5 marks)*

(c) Find, in terms of *g*, *l*, *m* and *U*, the tension in the string when the particle returns to *P*. *(2 marks)*

Turn over ▶

5 The graph shows a model for the resultant horizontal force on a car, which varies as it accelerates from rest for 20 seconds. The mass of the car is 1200 kg.

Force (N)
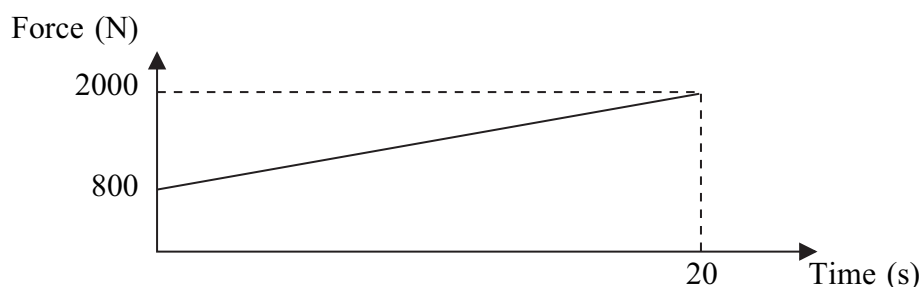
(a) The acceleration of the car at time t seconds is a m s^{-2}. Show that

$$a = \frac{2}{3} + \frac{t}{20}, \quad \text{for} \quad 0 \leqslant t \leqslant 20$$

(5 marks)

(b) Find an expression for the velocity of the car at time t. *(3 marks)*

(c) Find the distance travelled by the car in the 20 seconds. *(4 marks)*

(d) An alternative model assumes that the resultant force increases uniformly from 900 to 2100 newtons during the 20 seconds. Which term in your expression for the velocity would change as a result of this modification? Explain why. *(2 marks)*

6 A car of mass 1200 kg travels round a roundabout on a horizontal, circular path at a constant speed of $14 \, \text{m s}^{-1}$. The radius of the circle is 50 metres. Assume that there is no resistance to the motion of the car and that the car can be modelled as a particle.

(a) A friction force, directed towards the centre of the roundabout, acts on the car as it moves. Show that the magnitude of this friction force is 4704 N. *(4 marks)*

(b) The coefficient of friction between the car and the road is μ. Show that $\mu \geqslant 0.4$. *(3 marks)*

7 A particle of mass 20 kg moves along a straight horizontal line. At time t seconds the velocity of the particle is $v \, \text{m s}^{-1}$. A resistance force of magnitude $10\sqrt{v}$ newtons acts on the particle while it is moving. At time $t = 0$ the velocity of the particle is $25 \, \text{m s}^{-1}$.

(a) Show that, at time t

$$v = \left(\frac{20 - t}{4} \right)^2$$

(7 marks)

(b) State the value of t when the particle comes to rest. *(1 mark)*

END OF QUESTIONS